Soft breaking of BRST invariance for introducing non-perturbative infrared effects in a local and renormalizable way

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Abstract

The possibility of introducing non-perturbative infrared effects leading to a modification of the long distance behavior of gauge theories through a soft breaking of the $BRST$ invariance is investigated. The method reproduces the Gribov-Zwanziger action describing the restriction of the domain of integration in the Feynman path integral to the Gribov region and a model for the dynamical quark mass generation is presented. The soft symmetry breaking relies on the introduction of $BRST$ doublets and massive physical parameters, which allow one to distinguish the infrared region from the ultraviolet one, within the same theory.

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1 Introduction

The task of introducing non-perturbative effects is one of the most difficult challenges in quantum field theory. In this letter we propose to introduce non-perturbative infrared effects in a way that preserves the basic properties of continuum field theory, namely: locality and renormalizability. The idea relies on the possibility of introducing in a controllable way local terms which give rise to a soft breaking of the $BRST$ symmetry, meaning that the dimension of these breaking terms is smaller than the space-time dimension. Such soft terms can be neglected in the deep ultraviolet region, where one recovers the notion of exact $BRST$ invariance. However, in the low energy infrared region, the soft breaking terms cannot be neglected, and produce a quantitative modification of the large distance behavior of the theory. Furthermore, these terms are introduced in a way that preserves the nilpotency of the $BRST$ operator and doesn't modify its cohomology. As a consequence, the space of the local observables of the theory, identified with the cohomology classes of the $BRST$ operator in the space of local field polynomials, remains unchanged. In other words, assuming that the relevant correlation functions of the theory are of the type $\langle O_1(x_1)....O_n(x_n)\rangle$ with $O_1(x_1), ..., O_n(x_n)$ field polynomials belonging to the cohomology of the $BRST$ operator, we attempt at introducing nonperturbative effects which might modify the infrared behavior of $\langle O_1(x_1)....O_n(x_n)\rangle$, while leaving unchanged its behavior in the deep ultraviolet region. Thus, the same correlation function can display a different behavior, according to the region which is being considered. Our aim here is precisely that of keeping the same set of observables of the starting theory, while modifying the large distance behavior of their correlation functions.

The first step in order to achieve this goal is that of introducing a suitable set of additional fields, assembled as $BRST$ doublets. Such fields do not modify the cohomology of the $BRST$ operator. Thus, their introduction leaves unchanged the original content of observables of the theory. This follows from the fact that $BRST$ doublets always give rise to invariant terms which can be cast in the form of exact $BRST$ variations, thus yielding vanishing cohomology [1].

Secondly, these $BRST$ doublets are coupled to a set of dimensionfull parameters in such a way that the initial $BRST$ invariance of the theory can only be softly broken by terms which can be kept under control at the quantum level. This nontrivial requirement can be fulfilled thanks to the $BRST$ doublet structure of the fields coupled to these dimensionful parameters, which determine suitable linearly broken Ward identities, as well as generalized Slavnov–Taylor identities, ensuring the renormalizability and the stability of the theory in the presence of the breaking.

The last step is that of requiring that these parameters are not treated as free parameters of the theory. Instead, they are determined in a self-consistent way at the quantum level by imposing appropriate gap equations, reflecting the possible non-perturbative condensation of the local operators coupled to them. This allows one to express them in terms of the original coupling constant g of the theory, exhibiting the typical non-perturbative behavior $\sim e^{-\frac{1}{g^2}}$. Albeit we cannot specify in full generality the explicit form of these gap equations, it is worth underlining that the dimension of the operators coupled to these parameters is smaller than the space-time dimensions. The analytic study of the possible condensation of these operators is simplified as compared to the case of an operator of maximum dimension. In fact, several techniques are nowadays at our disposal in order to face the study of the condensation of local operators of lower dimension: effective potential, local composite operator technique, the renormalization group equations, variational principles as the minimal sensitivity principle, the fastest apparent convergence criterion, etc. Let us mention, for example, the case of the condensation of the dimension two mass operator $\left\langle\left(\frac{1}{2}A^2 - \alpha\bar{c}c\right)\right\rangle$ in the Yang–Mills theory expressed in the maximal Abelian gauge. The condensation of this operator, see [2] and references therein, provides a non-perturbative dynamical mass for off-diagonal gluons, a feature of great relevance for the Abelian dominance hypothesis for the dual

superconductivity picture of color confinement.

One can certainly question the meaning of introducing a soft breaking of the $BRST$ symmetry, since the latter is known to be related to physical properties of the theory, such as the unitarity and the characterization of the physical subspace. The justification is for models displaying non-perturbative phases, which require a better knowledge of the infrared sector. This is the case, for example, of the QCD confining non-abelian gauge theories. As is well known, the particle interpretation of QCD in terms of partons - gluons and quarks- is lost in the low energy region. Unitarity cannot and shouldn't be defined for the gluon and quark sector in the infrared region, where confinement sets in, and gluons and quarks do not correspond anymore to the physical excitations of the theory. Rather, the physical states are given by colorless bound states like baryons, mesons and glueballs. Moreover, the framework that we introduce here can also include the description of non-perturbative long distance effects in supersymmetric as well as in topological theories, both by taking into account the effect of the Gribov phenomenon and a soft breaking of the supersymmetry, as in the Seiberg–Witten theory [3].

As a last remark, we would like to underline that the soft breaking of the $BRST$ symmetry introduced here is meant to be an explicit breaking, as opposed to the possibility of achieving a spontaneous symmetry breaking. Indeed, the concept of spontaneous $BRST$ symmetry breaking would imply the existence of Goldstone massless modes, which would give rise to a rather different framework.

The paper is organized as follows. In Sect.2 we give an overview of the introduction of the $BRST$ doublet fields and of the related dimensionfull parameters. We point out that the presence of the soft breaking of the $BRST$ invariance ensures that the aforementioned dimensionfull parameters are physical parameters of the theory and not unphysical ones, as it would be the case of gauge parameters. An elementary algebraic proof of this property is given. Thus, in the infrared domain, the values of the correlation functions of the theory will explicitly depend on these parameters, which would themselves be related to Λ_{QCD} by generalized gap equations. In Sect.3, we show that the method yields the so-called Gribov-Zwanziger action which implements the restriction of the domain of integration in the path integral to the Gribov region, plus a relevant soft term. In Sect.4 we present a model for the dynamical quark mass generation.

2 Soft breaking of the BRST invariance

We start by considering a quantized action $S_{\rm inv}(\phi)$, endowed with a set of fields generically denoted by ϕ, with the following properties:

- the action exhibits exact $BRST$ invariance
$$sS_{\rm inv}(\phi) = 0 \ , \tag{1}$$
where s denotes the nilpotent $BRST$ operator
$$s^2 = 0 \ , \tag{2}$$
- $S_{\rm inv}$ is multiplicatively renormalizable leading, in particular, to a consistent perturbation theory in the ultraviolet region.

A typical example of such an action is provided by non-abelian $SU(N)$ Euclidean Yang-Mills theories, namely
$$S_{\rm inv} = S_{YM} + S_{gf} \ , \tag{3}$$

where S_{YM} stands for the Yang–Mills action

$$S_{YM} = \frac{1}{4} \int d^4x \, F^a_{\mu\nu} F^a_{\mu\nu} \; , \tag{4}$$

and S_{gf} is the gauge-fixing term of the Landau gauge

$$S_{gf} = \int d^4x \, \left(ib^a \partial_\mu A^a_\mu + \overline{c}^a \partial_\mu D^{ab}_\mu c^b \right) \; , \tag{5}$$

with b^a being the Lagrange multiplier field enforcing the Landau condition, $\partial_\mu A^a_\mu = 0$, and $\left(\overline{c}^a, c^b\right)$ the Faddeev-popov ghosts. As required, expression (3) is renormalizable, while displaying $BRST$ invariance [1].

We will now extend this local action for tentatively introducing non-perturbative infrared effects within the context of local quantum field theory, which will lead to a modification of the long distance behavior of the theory, while leaving unaffected its ultraviolet behavior. To do so, we introduce a set of fields, generically denoted as $\left(\underline{\alpha}, \underline{\beta}\right)$, which transform as $BRST$ doublets

$$\begin{aligned} s\underline{\alpha} &= \underline{\beta} \, , \\ s\underline{\beta} &= 0 \, . \end{aligned} \tag{6}$$

Such fields give rise to a trivial cohomology, meaning that any local $BRST$ invariant functional $F(\underline{\alpha}, \underline{\beta})$ has necessarily the form of an exact $BRST$ term

$$sF(\underline{\alpha}, \underline{\beta}) = 0 \;\Rightarrow\; F(\underline{\alpha}, \underline{\beta}) = s\widehat{F}(\underline{\alpha}, \underline{\beta}) \; , \tag{7}$$

for some local $\widehat{F}(\underline{\alpha}, \underline{\beta})$.

After the introduction of the fields $(\underline{\alpha}, \underline{\beta})$ and of a corresponding $BRST$ invariant term describing their propagation,

$$S^{\alpha\beta}_{\rm inv}(\underline{\alpha}, \underline{\beta}) = s\widehat{S}(\underline{\alpha}, \underline{\beta}) \tag{8}$$

one defines a set of dimensionfull parameters $(\underline{\sigma})$ and a local term $S_\sigma(\underline{\sigma}, \underline{\alpha}, \underline{\beta}, \phi)$ which gives rise to a soft breaking of the $BRST$ invariance, as expressed by

$$s\left(S_{\rm inv}(\phi) + S^{\alpha\beta}_{\rm inv}(\underline{\alpha}, \underline{\beta}) + S_\sigma(\underline{\sigma}, \underline{\alpha}, \underline{\beta}, \phi)\right) = sS_\sigma = \underline{\sigma}\, \Delta(\underline{\alpha}, \underline{\beta}, \phi) \; . \tag{9}$$

Since $(\underline{\sigma})$ are dimensionfull parameters, the breaking term $\Delta(\underline{\alpha}, \underline{\beta}, \phi)$ is an integrated local field polynomial whose dimension is smaller that the space-time dimension, *i.e.* it is a soft breaking. As such, it can be neglected in the deep ultraviolet region. Moreover, taking into account that the additional fields $(\underline{\alpha}, \underline{\beta})$ form $BRST$ doublets, it follows that the ultraviolet behavior of the model is left unmodified.

The introduction of the soft parameters $(\underline{\sigma})$ has to be done in a way that preserves the multiplicative renormalizability of the modified theory. This physical requirement allows one to carry out calculations in a consistent way. Furthermore, it puts severe constraints on the way the soft parameters are introduced. The simplest and consistent way is to couple the additional doublet fields $\left(\underline{\alpha}, \underline{\beta}\right)$ linearly to the original fields ϕ, so that the resulting breaking term is a quadratic term in the fields. As a consequence, the propagator of the ϕ-field gets modified in the infrared by the soft parameters entering the quadratic coupling. In turn, this will affect the infrared behavior of the correlation functions of the theory, as can be seen, in particular, from the modifications at small momentum that are brought to the propagators of partons. Coupling the fields in that way has important consequences on the Ward identities of the theory. In fact, a quadratic term means that the additional fields $\left(\underline{\alpha}, \underline{\beta}\right)$ couple linearly to the original fields ϕ.

As we shall see in the examples discussed in the next sections, this feature enables one to write down a suitable set of linearly broken Ward identities, controlling the renormalizability of the theory. In summary, the introduction of the infrared soft breaking can be done in a way in which the resulting action is stable against quantum corrections. This will ensure that no additional parameters have to be introduced.

The presence of the breaking term $\Delta(\underline{\alpha}, \underline{\beta}, \phi)$ enables one to establish that the dimensionfull parameters $(\underline{\sigma})$ are physical parameters of the theory, as for instance the Gribov parameter γ [4, 5] of the Gribov-Zwanziger action. In fact, taking the derivative of both sides of eq.(9) with respect to $\underline{\sigma}$, one gets

$$s \frac{\partial S_\sigma}{\partial \underline{\sigma}} = \Delta \ , \tag{10}$$

from which it is apparent that $\frac{\partial S_\sigma}{\partial \underline{\sigma}}$ cannot be cast in the form of an exact $BRST$ variation, namely

$$\frac{\partial S_\sigma}{\partial \underline{\sigma}} \neq s\widehat{\Delta} \ , \tag{11}$$

for some local $\widehat{\Delta}$. Equation (11) ensures that the parameters $(\underline{\sigma})$ are physical parameters of the theory. As such, they will influence the expressions of the correlation functions of the operators belonging to the cohomology os s, modifying their long distance behavior. We see therefore that the soft breaking term Δ plays a key role in order to introduce nontrivial parameters. Suppose in fact that, instead of giving rise to a soft breaking, the term $S_\sigma(\underline{\sigma}, \underline{\alpha}, \underline{\beta}, \phi)$ would be left invariant by the $BRST$ operator, *i.e.*

$$sS_\sigma(\underline{\sigma}, \underline{\alpha}, \underline{\beta}, \phi) = 0 \ . \tag{12}$$

Therefore, owing to the doublet structure of $(\underline{\alpha}, \underline{\beta})$, a local functional $\widehat{S}_\sigma$ should exist such that

$$S_\sigma(\underline{\sigma}, \underline{\alpha}, \underline{\beta}, \phi) = s\widehat{S}_\sigma \ , \tag{13}$$

from which it would follow that

$$\frac{\partial S_\sigma}{\partial \underline{\sigma}} = s \frac{\partial \widehat{S}_\sigma}{\partial \underline{\sigma}} \ , \tag{14}$$

which would imply that $(\underline{\sigma})$ would be unphysical parameters. As a consequence, the correlation functions of the observables of the theory would be independent from $(\underline{\sigma})$, meaning that the introduction of these parameters would have no effects on the theory.

3 The example of the Gribov-Zwanziger Lagrangian

As a first example, we give a derivation of a refined version of the Gribov-Zwanziger Lagrangian, see [4, 5], within the previous set up. We start with the Yang–Mills action, as given in eq.(3), and we add a set of fields $(\overline{\varphi}^{ac}_\mu, \varphi^{ac}_\mu)$, $(\overline{\omega}^{ac}_\mu, \omega^{ac}_\mu)$ transforming as $BRST$ doublets. For the $BRST$ invariant action we write

$$S_0 = S_{YM} + S_{gf} \ + s\int d^4x \ \left(\overline{\omega}^{ac}_\mu \partial_\nu \left(\partial_\nu \varphi^{ac}_\mu + gf^{abm} A^b_\nu \varphi^{mc}_\mu \right) - \mu^2 \overline{\omega}^{ac}_\mu \varphi^{ac}_\mu \right) \ , \tag{15}$$

with

$$\begin{aligned} sA^a_\mu &= -(D_\mu c)^a \ , \\ sc^a &= \frac{1}{2} g f^{abc} c^b c^c \ , \\ s\overline{c}^a &= ib^a \ , \qquad sb^a = 0 \ , \end{aligned} \tag{16}$$

and

$$
\begin{aligned}
s\varphi_\mu^{ac} &= \omega_\mu^{ac} \ , \qquad s\omega_\mu^{ac} = 0 \ , \\
s\overline{\omega}_\mu^{ac} &= \overline{\varphi}_\mu^{ac} \ , \qquad s\overline{\varphi}_\mu^{ac} = 0 \ ,
\end{aligned} \tag{17}
$$

$$
sS_0 = 0 \ . \tag{18}
$$

For the explicit expression of S_0 one gets

$$
\begin{aligned}
S_0 &= S_{\mathrm{YM}} + \int d^4x \ \left(ib^a \partial_\mu A_\mu^a + \overline{c}^a \partial_\mu \left(D_\mu c\right)^a \right) \\
&+ \int d^4x \left(\overline{\varphi}_\mu^{ac} \partial_\nu \left(\partial_\nu \varphi_\mu^{ac} + g f^{abm} A_\nu^b \varphi_\mu^{mc} \right) - \overline{\omega}_\mu^{ac} \partial_\nu \left(\partial_\nu \omega_\mu^{ac} + g f^{abm} A_\nu^b \omega_\mu^{mc} \right) \right) \\
&- \int d^4x \ \left(g \left(\partial_\nu \overline{\omega}_\mu^{ac} \right) f^{abm} \left(D_\nu c \right)^b \varphi_\mu^{mc} + \mu^2 \left(\overline{\varphi}_\mu^{ac} \varphi_\mu^{ac} - \overline{\omega}_\mu^{ac} \omega_\mu^{ac} \right) \right) \ .
\end{aligned} \tag{19}
$$

The fields $\left(\overline{\varphi}_\mu^{ac}, \varphi_\mu^{ac}\right)$ are a pair of complex conjugate bosonic fields. They have dimension one and zero ghost number. Each field has $4\left(N^2-1\right)^2$ components. Similarly, the fields $\left(\overline{\omega}_\mu^{ac}, \omega_\mu^{ac}\right)$ are anticommuting, having ghost number $(-1, 1)$, respectively.

From the $BRST$ doublet structure of the additional fields $\left(\overline{\varphi}_\mu^{ac}, \varphi_\mu^{ac}, \overline{\omega}_\mu^{ac}, \omega_\mu^{ac}\right)$ it follows that the action S_0 is equivalent to pure Yang–Mills theory, leading to the same ultraviolet behavior. The doublet structure implies in fact that there is an exact compensation in the Feynman diagrams among the bosonic sector $\left(\overline{\varphi}_\mu^{ac}, \varphi_\mu^{ac}\right)$ and the anticommuting one $\left(\overline{\omega}_\mu^{ac}, \omega_\mu^{ac}\right)$. Also, it is easily verified that the integration over the auxiliary fields $\left(\overline{\varphi}_\mu^{ac}, \varphi_\mu^{ac}, \overline{\omega}_\mu^{ac}, \omega_\mu^{ac}\right)$ amounts to introduce a unity in the partition function, meaning that the physical content of the theory is precisely the same as that of the Faddeev-Popov action.

We now introduce a soft breaking of the $BRST$ symmetry in a way which modifies the low energy sector, while keeping the original ultraviolet behavior as well as the renormalizability of the theory, as explained in the last section. We thus introduce a quadratic term describing the coupling among the gauge field A_μ^a and the bosonic fields $\left(\overline{\varphi}_\mu^{ac}, \varphi_\mu^{ac}\right)$. This term will affect the gluon propagator $\langle A_\mu^a(k) A_\nu^b(-k) \rangle$ in the infrared. As a consequence, the correlation functions of the gauge invariant quantities, like e.g. $\left\langle F^2(x) F^2(y) \right\rangle$, will get modified in the low energy region. For this quadratic term, one can write the following expression

$$
S_\gamma = -\gamma^2 g \int d^4x \ \left(f^{abc} A_\mu^a \varphi_\mu^{bc} + f^{abc} A_\mu^a \overline{\varphi}_\mu^{bc} \right) \ . \tag{20}
$$

where γ is a parameter with mass dimension one. With the introduction of the term S_γ the action

$$
S = S_0 + S_\gamma \tag{21}
$$

does not display anymore exact $BRST$ invariance. Instead, one has the softly broken identity.

$$
sS = \gamma^2 \Delta_\gamma \ , \tag{22}
$$

where the soft breaking term is given by

$$
\Delta_\gamma = \int d^4x \, g f^{abc} \left(\left(D_\mu^{am} c^m \right) \left(\varphi_\mu^{bc} + \overline{\varphi}_\mu^{bc} \right) - A_\mu^a \omega_\mu^{bc} \right) \ . \tag{23}
$$

Looking at the gluon propagator $\langle A_\mu^a(k) A_\nu^b(-k) \rangle$, one finds

$$
\langle A_\mu^a(k) A_\nu^b(-k) \rangle = \delta^{ab} \frac{k^2 + \mu^2}{k^4 + \mu^2 k^2 + \gamma^4} \ \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \tag{24}
$$

or

$$\langle A^a_\mu(k)A^b_\nu(-k)\rangle = \delta^{ab}\frac{1}{k^2+\mathcal{M}^2(k^2)}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right) \tag{25}$$

with

$$\mathcal{M}^2(k^2) = \frac{\gamma^4}{k^2+\mu^2} \ . \tag{26}$$

We see therefore that, while keeping the usual ultraviolet behavior $\simeq 1/k^2$ at very high momenta, expression (24) turns out to be deeply modified in the infrared region $k \approx 0$ by the presence of the soft parameters (γ,μ). The propagator (24) is not of the Gribov type, as it does not vanish at the origin, thanks to the presence of the parameter μ [4, 5]. Remarkably, this propagator is in good agreement with the most recent lattice numerical simulations [6, 7, 8, 9]. In particular, it exhibits violation of reflection positivity, a feature which is usually interpreted a signal for gluon confinement.

Without entering into details, it is worth mentioning here that the action S of expression (21) has a profound geometrical meaning [10, 11], as it allows to restrict the domain of integration in the path integral to the so called Gribov region Ω, defined as the set of gauge fields fulfilling the Landau condition and for which the Faddeev-Popov operator $\mathcal{M}^{ab} = -\partial_\mu D^{ab}_\mu$ is strictly positive, namely

$$\Omega = \left\{ A^a_\mu;\ \partial_\mu A^a_\mu = 0, \quad \mathcal{M}^{ab} = -\partial_\mu D^{ab}_\mu > 0 \right\} \ . \tag{27}$$

The restriction to this region is needed in order to properly take into account the existence of the Gribov copies. In particular, the parameter γ, known as the Gribov parameter, is not a free parameter of the theory, being determined in a self-consistent way through the gap equation [10, 11]

$$\frac{\delta\Gamma}{\delta\gamma^2} = 0 \ , \tag{28}$$

where Γ stands for the $1PI$ generating functional. This equation enables us to express it in terms of the coupling constant g and of the non-perturbative invariant scale Λ_{QCD}. In much the same way, the parameter μ has been determined by means of a variational principle, see refs.[4, 5] for the numerical characterization of (γ,μ). It is worth mentioning that the introduction of the $BRST$ invariant mass term in expression (19)

$$\int d^4x\ \mu^2\left(\overline{\varphi}^{ac}_\mu\varphi^{ac}_\mu - \overline{\omega}^{ac}_\mu\omega^{ac}_\mu\right) \ . \tag{29}$$

follows from the observation that the dimension two condensate $\left\langle\overline{\varphi}^{ac}_\mu\varphi^{ac}_\mu - \overline{\omega}^{ac}_\mu\omega^{ac}_\mu\right\rangle$ has a non-vanishing value. In fact, as shown in [4, 5], one finds

$$\begin{aligned}\left\langle\overline{\varphi}^{ac}_\mu\varphi^{ac}_\mu - \overline{\omega}^{ac}_\mu\omega^{ac}_\mu\right\rangle &= \frac{3(N^2-1)}{64\pi}\lambda \\ \lambda^4 &= 2g^2N\gamma^4 \ ,\end{aligned} \tag{30}$$

from which one sees that $\left\langle\overline{\varphi}^{ac}_\mu\varphi^{ac}_\mu - \overline{\omega}^{ac}_\mu\omega^{ac}_\mu\right\rangle$ is nonzero for non-vanishing Gribov parameter γ. Let us also mention that the appearance of the $BRST$ soft breaking term Δ_γ can be traced back to the properties of the Gribov region. It arises as a consequence of the fact that any infinitesimal gauge transformation of a field configuration belonging to Ω gives rise to a configuration which lies outside of Ω [5].

A remarkable aspect of the action (S_0+S_γ) is its multiplicative renormalizability[1] [10, 11, 12, 4, 5]. Only two independent renormalization factors are needed to account for all ultraviolet divergences. It

[1] We remind here that vacuum terms of the kind $\int d^4x 4(N^2-1)\gamma^4$ have to be properly taken into account when solving the gap equation (28). These terms show up in fact during the renormalization procedure, as it follows by employing the method of the external sources, as outlined in [10, 11, 12, 4, 5].

should be noticed that both the Gribov parameter γ and the dynamical mass μ do not renormalize independently, the corresponding renormalization factors being given by suitable products of the two independent renormalization factors of the theory which can be chosen as being the renormalization of the coupling constant Z_g and of the gluon field Z_A. The soft breaking Δ_γ does not invalidate the existence of the Slavnov–Taylor identities for the renormalized $1PI$ generating functional Γ, namely

$$\mathcal{S}(\Gamma) = [\Delta_\gamma \cdot \Gamma] \ , \tag{31}$$

where $[\Delta_\gamma \cdot \Gamma]$ is the generator of the $1PI$ Green functions with the insertion of the breaking Δ_γ and

$$\mathcal{S}(\Gamma) = \int d^4x \left(\frac{\delta\Gamma}{\delta A^a_\mu} \frac{\delta\Gamma}{\delta \Omega^a_\mu} + \frac{\delta\Gamma}{\delta c^a} \frac{\delta\Gamma}{\delta L^a} + b^a \frac{\delta\Gamma}{\delta \overline{c}^a} + \omega^{ac}_\mu \frac{\delta\Gamma}{\delta \varphi^{ac}_\mu} + \overline{\varphi}^{ac}_\mu \frac{\delta\Gamma}{\delta \overline{\omega}^{ac}_\mu} \right) \tag{32}$$

with Ω^a_μ and L^a being the external sources coupled to the nonlinear $BRST$ transformation of the gluon field A^a_μ and of the ghost field c^a. Equation (31) is the quantum generalization of the broken identity (22). It is worth emphasizing that the Slavnov–Taylor identities (31) have in fact a powerful predictive character. They allow us to establish the relationships among the various Green functions of the theory in a way which takes into account the presence of the Gribov horizon [5].

Let us also mention that, due to the presence of the soft breaking, the mass parameter μ^2 acquires the meaning of a physical parameter of the theory, even if it has been introduced through an s-exact term, eq.(15). This important feature has a simple understanding by noticing that, in the absence of the Gribov parameter, *i.e.* $\gamma = 0$, the contributions of the two $BRST$ doublets, $\left(\overline{\varphi}^{ac}_\mu, \varphi^{ac}_\mu\right)$ and $\left(\overline{\omega}^{ac}_\mu, \omega^{ac}_\mu\right)$, compensate each other in an exact way. In fact, setting $\gamma = 0$, one immediately realizes that the gluon propagator in eq.(24) gets independent from the parameter μ^2. However, in the presence of the breaking, *i.e.* for $\gamma \neq 0$, the compensation among the two sectors $\left(\overline{\varphi}^{ac}_\mu, \varphi^{ac}_\mu\right)$ and $\left(\overline{\omega}^{ac}_\mu, \omega^{ac}_\mu\right)$ does not hold anymore, due to the fact that the fields $\left(\overline{\varphi}^{ac}_\mu, \varphi^{ac}_\mu\right)$ have been coupled to the gluon field through the quadratic term of eq.(20). As a consequence, the parameter μ^2 enters in a nontrivial way the gluon propagator as well as the correlation functions of the theory. A more formal proof of this property can be given by making use of the renormalized Slavnov-Taylor identities (31).

4 A model for the dynamical quark mass generation

As second example, we present a possible mechanism for introducing a dynamical quark mass. To some extent, this is a way to push further the gate opened by Gribov-Zwanziger in the gluon sector, using the idea that all partons of a confining and asymptotically free theory should get a propagator with a momentum dependence as in eq.(24), since they all have very analogous physical status. The mechanism will consist in introducing $BRST$ doublets in correspondence with the quarks, and check the possible $BRST$ symmetry breaking soft terms that are allowed by power counting.

We thus supplement the Yang–Mills action of expression (3) by the fermionic matter term

$$S_\psi = \int d^4x \ \overline{\psi} \gamma_\mu D_\mu \psi \ , \tag{33}$$

with

$$\begin{aligned} s\psi &= c^a T^a \psi \ , \\ sS_\psi &= 0 \ . \end{aligned} \tag{34}$$

Further, we introduce two $BRST$ doublets of spinor fields, (φ_F, ω_F) and (λ_F, η_F) transforming as

$$
\begin{aligned}
s\varphi_F &= \omega_F \,, \qquad s\omega_F = 0 \,, \\
s\eta_F &= \lambda_F \,, \qquad s\lambda_F = 0 \,.
\end{aligned} \tag{35}
$$

The fields (φ_F, ω_F) have dimension 1/2 and ghost number $(0, 1)$, respectively, while (λ_F, η_F) have dimension 3/2 and ghost number $(0, -1)$.

According to the set-up of Sect.2, the invariant action describing the propagation of these fields is obtained from an exact $BRST$ variation, namely

$$
\begin{aligned}
S_{\varphi\lambda} &= s \int d^4x \left(-\overline{\eta}_F \partial^2 \varphi_F + \overline{\varphi}_F \partial^2 \eta_F + m^2 (\overline{\eta}_F \varphi_F - \overline{\varphi}_F \eta_F) \right) \\
&= \int d^4x \left(-\overline{\lambda}_F \partial^2 \varphi_F - \overline{\varphi}_F \partial^2 \lambda_F - \overline{\eta}_F \partial^2 \omega_F + \overline{\omega}_F \partial^2 \eta_F \right. \\
&\qquad \left. + m^2 (\overline{\lambda}_F \varphi_F + \overline{\varphi}_F \lambda_F + \overline{\eta}_F \omega_F - \overline{\omega}_F \eta_F) \right) \,.
\end{aligned} \tag{36}
$$

For the soft breaking term of the $BRST$ symmetry we get

$$
S_M = M_1^2 \int d^4x \left(\overline{\varphi}_F \psi + \overline{\psi} \varphi_F \right) - M_2 \int d^4x \left(\overline{\lambda}_F \psi + \overline{\psi} \lambda_F \right) \,. \tag{37}
$$

From expression (37) one realizes that, in a way similar to the case of the Gribov-Zwanziger action, the additional fields φ_F, λ_F couple linearly to the matter field ψ. As a consequence, the propagator $\langle \psi(k) \overline{\psi}(-k) \rangle$ of the matter field will get deeply modified in the infrared by the presence of the soft mass parameters m, M_1, M_2. In fact, one finds

$$
\langle \psi(k) \overline{\psi}(-k) \rangle = \frac{i\gamma_\mu k_\mu + \mathcal{A}(k)}{k^2 + \mathcal{A}^2(k)} \,, \tag{38}
$$

where the mass function $\mathcal{A}(k)$ is given by

$$
\mathcal{A}(k) = \frac{2M_1^2 M_2}{k^2 + m^2} \,. \tag{39}
$$

The function $\mathcal{A}(k)$ is completely analogous to the function $\mathcal{M}^2(k^2)$, which has appeared in the propagator for the gluon in eq.(25). Its geometrical interpretation is however lacking, since the Gribov phenomenon is left untouched by the quark dependance. For the quarks, the demand of a modification of the propagator is thus motivated by their confinement and by the chiral symmetry breaking, and it is made possible in quantum field theory by this general mechanism involving the introduction of soft breaking terms using doublets.

Interestingly, expression (39) provides a good fit for the dynamical quark mass in the infrared region, as reported by the lattice numerical simulations of the quark propagator in the Landau gauge, see for instance [13, 14].

Although being out of the main goal of the present letter, let us mention that the renormalizability of the spinor action $(S_{\varphi\lambda} + S_M)$ stems from the fact that the fields (φ_F, λ_F) , (ω_F, η_F) are introduced in a way which enables us to write a local set of linearly broken identities, as expressed by

$$
\frac{\delta \left(S_{\varphi\lambda} + S_M \right)}{\delta \overline{\eta}_F} = -\partial^2 \omega_F + m^2 \omega_F \,, \tag{40}
$$

$$
\frac{\delta \left(S_{\varphi\lambda} + S_M \right)}{\delta \overline{\omega}_F} = \partial^2 \eta_F - m^2 \eta_F \,, \tag{41}
$$

$$\frac{\delta\left(S_{\varphi\lambda}+S_{M}\right)}{\delta\overline{\lambda}_{F}} = -\partial^{2}\varphi_{F}+m^{2}\varphi_{F}-M_{2}\psi \ , \tag{42}$$

$$\frac{\delta\left(S_{\varphi\lambda}+S_{M}\right)}{\delta\overline{\varphi}_{F}} = -\partial^{2}\lambda_{F}+m^{2}\lambda_{F}+M_{1}^{2}\psi \ . \tag{43}$$

Thanks to the fact that the right hand sides of eqs.$(40)-(43)$ is linear in the fields, it turns out that these equations can be converted in a set of powerful Ward identities which constrain very much the possible counterterms which can show up at quantum level, ensuring that the action $(S_{\varphi\lambda}+S_{M})$ is stable against quantum corrections. The reader might have in fact remarked that equations $(40)-(43)$ follow from the fact that the quadratic terms in expression (36) contain simple space-time derivatives, e.g. $\overline{\eta}_{F}\partial^{2}\varphi_{F}$, and not covariant derivatives of the kind $\overline{\eta}_{F}D^{2}\varphi_{F}$. This term would jeopardize the linearity in the quantum fields of eqs.$(40)-(43)$, as quadratic and cubic terms in the fields would be present. These terms have to be treated as composite operators, requiring a proper renormalization procedure. In summary, eqs.$(40)-(43)$ would loose their usefulness. Moreover, as much as in the case of a $\phi^{4}-$theory, the presence of a term of the kind $\overline{\eta}_{F}D^{2}\varphi_{F}$ would require the introduction of a quartic spinor interaction like $\rho\left(\overline{\eta}_{F}\varphi_{F}\right)\left(\overline{\varphi}_{F}\eta_{F}\right)$ with its own dimensionless parameter ρ, implying that we would be leaving the space of soft parameters. Let us finally mention that a similar set of Ward identities can be derived in the case of the Gribov-Zwanziger action [10, 11, 12, 4, 5].

5 Conclusion

In this letter we have illustrated a proposal in order to introduce non-perturbative infrared effects leading to a modification of the long distance behavior of the correlation functions of the theory.

The whole idea relies on the introduction of additional fields giving rise to $BRST$ doublets. These fields do not modify the set of original observables of the theory, identified here with the cohomology classes of the $BRST$ operator. Subsequently, a set of soft parameters are introduced through terms which give rise to a soft breaking of the $BRST$ invariance. The simplest way to do this is by demanding that the doublet fields couple linearly to the original fields of the theory, so that the soft breaking is given by terms quadratic in the fields. As a consequence, the propagators of the original fields of the theory get deeply modified in the infrared. In turn, the correlation functions of the local observables of the theory gets modified too in the low energy region.

We have pointed out that such kind of coupling enables us to write down suitably linearly broken Ward identities which can ensure that the theory remains renormalizable in the presence of the breaking. Moreover, we have seen that the presence of this breaking allows us to give an elementary algebraic proof of the fact that the corresponding soft parameters are physical parameters and not unphysical ones.

The Gribov-Zwanziger action implementing the restriction in the Feynman path integral to the Gribov region Ω has been revised within the present set up. Finally, we have proposed a model which might be of a certain interest in the study of the dynamical quark mass generation. The details of the renormalizability of this model as well as a discussion of possible gap equations allowing to give an estimate of the parameters m, M_{1}, M_{2} will be the object of a more extended work [15].

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